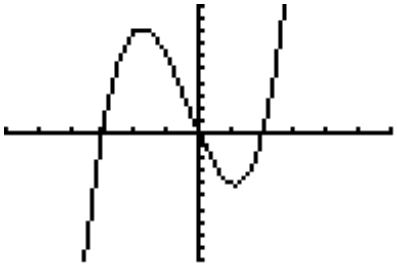
Worksheet #6

1. Consider $f(x) = x(x+3)(x-2)$ [-6, 6, 1] by [-10, 10,1]



Find the x- intercepts.

$0 = x(x+3)(x-2)$

$x = 0 \quad x+3 = 0 \quad x-2 = 0$

$x = 0 \quad x = -3 \quad x = 2$

Notice each of these x-intercepts occurs once.
Consider the graph; it crosses the x-axis at 0, -3 and 2.

$f(x) = x(x+3)(x-2)$ can also be written as $f(x) = x^3 + x^2 - 6x$

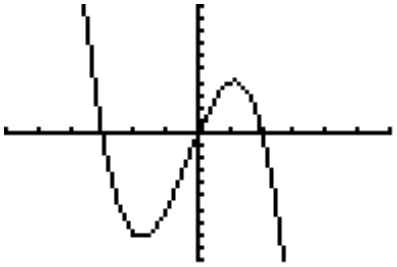
Degree is _____ which is **ODD** **EVEN**

Leading Coefficient is _____ which is **POSITIVE** **NEGATIVE**

End behavior: as $x \to -\infty, f(x) \to$ _____ ∞ $-\infty$ as $x \to \infty, f(x) \to$ _____ ∞ $-\infty$

Left-hand side f goes ↑ f goes ↓ *Right-hand side* f goes ↑ f goes ↓

2. Consider $f(x) = -x(x+3)(x-2)$ [-6, 6, 1] by [-10, 10, 1]



Find the x- intercepts.

$0 = -x(x+3)(x-2)$

$-x = 0 \quad x+3 = 0 \quad x-2 = 0$

$x = 0 \quad x = -3 \quad x = 2$

Notice each of these x-intercepts occurs once.
Consider the graph; it crosses the x-axis at 0, -3 and 2.

$f(x) = -x(x+3)(x-2)$ can also be written as $f(x) = -x^3 - x^2 + 6x$

Degree is _____ which is **ODD** **EVEN**

Leading Coefficient is _____ which is **POSITIVE** **NEGATIVE**

End behavior: as $x \to -\infty, f(x) \to$ _____ ∞ $-\infty$ as $x \to \infty, f(x) \to$ _____ ∞ $-\infty$

Left-hand side f goes ↑ f goes ↓ *Right-hand side* f goes ↑ f goes ↓

3. Consider $f(x) = x(x+3)^2(x-2)$ [-6, 6, 1] by [-20, 20, 2]

Find the x- intercepts.

$0 = x(x+3)^2(x-2)$

$x = 0 \quad x+3 = 0 \quad x+3 = 0 \quad x-2 = 0$

$x = 0 \quad x = -3 \quad x = -3 \quad x = 2$

Notice the x-intercepts of 0 and 2 each occur once - the graph of f crosses the x-axis at 0 and 2.

The x-intercept of -3 occurs twice - the graph of f touches the x-axis at -3.

$f(x) = x(x+3)^2(x-2)$ can also be written as $f(x) = x^4 + 4x^3 - 3x^2 - 18x$

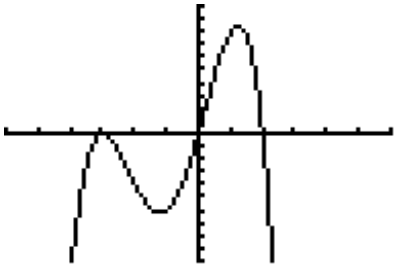
Degree is _____ which is **ODD** **EVEN**

Leading Coefficient is _____ which is **POSITIVE** **NEGATIVE**

End behavior: as $x \to -\infty, f(x) \to$ _____ ∞ $-\infty$ as $x \to \infty, f(x) \to$ _____ ∞ $-\infty$

Left-hand side f goes ↑ f goes ↓ *Right-hand side* f goes ↑ f goes ↓

4. Consider $f(x) = -x(x+3)^2(x-2)$ [-6, 6, 1] by [-20, 20, 2]



Find the x- intercepts.

$0 = -x(x+3)^2(x-2)$

$-x = 0 \quad x+3 = 0 \quad x+3 = 0 \quad x-2 = 0$

$x = 0 \quad x = -3 \quad x = -3 \quad x = 2$

Notice the x-intercepts of 0 and 2 each occur once - the graph of f crosses the x-axis at 0 and 2.

The x-intercept of -3 occurs twice - the graph of f touches the x-axis at this x-intercept.

$f(x) = -x(x+3)^2(x-2)$ can also be written as $f(x) = -x^4 - 4x^3 + 3x^2 + 18x$

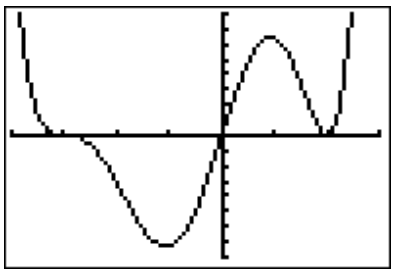
Degree is _____ which is **ODD** **EVEN**

Leading Coefficient is _____ which is **POSITIVE** **NEGATIVE**

End behavior: as $x \to -\infty, f(x) \to$ _____ ∞ $-\infty$ as $x \to \infty, f(x) \to$ _____ ∞ $-\infty$

Left-hand side f goes ↑ f goes ↓ *Right-hand side* f goes ↑ f goes ↓

5. Consider $f(x) = x(x+3)^3(x-2)^2$ [-4, 3, 1] by [-80, 80,10]



Find the x-intercepts.

$0 = x(x+3)^3(x-2)^2$

$x = 0,\ x+3 = 0,\ x+3 = 0,\ x+3 = 0,\ x-2 = 0,\ x-2 = 0$

$x = 0,\quad x = -3,\quad x = -3,\quad x = -3,\quad x = 2\quad x = 2$

Notice the x-intercept of 0 occurs once - the graph of f crosses the x-axis at 0.

The x-intercept of -3 occurs three times the graph of f crosses the x-axis at -3.

The x-intercept of 2 occurs twice - the graph of f touches the x-axis at 2.

$f(x) = x(x+3)^3(x-2)^2$ can also be written as $f(x) = x^6 + 5x^5 - 5x^4 - 45x^3 + 108x$

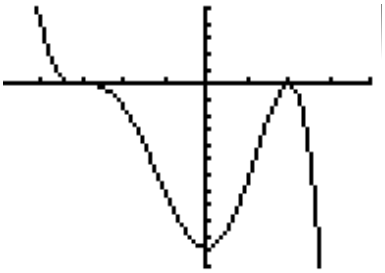
Degree is _____ which is **ODD** **EVEN**

Leading Coefficient is _____ which is **POSITIVE** **NEGATIVE**

End behavior: as $x \to -\infty$, $f(x) \to$ _____ ∞ $-\infty$ as $x \to \infty$, $f(x) \to$ _____ ∞ $-\infty$

Left-hand side f goes ↑ f goes ↓ *Right-hand side* f goes ↑ f goes ↓

6. Consider $f(x) = -(x+3)^3(x-2)^2$ [-5, 4, 1] by [-120, 50, 10]



Find the x- intercepts.

$0 = -(x+3)^3(x-2)^2$

$0 = (x+3)^3(x-2)^2$

$x+3 = 0,\ x+3 = 0,\ x+3 = 0,\ x-2 = 0,\ x-2 = 0$

$x = -3,\quad x = -3,\quad x = -3,\quad x = 2\quad x = 2$

The x-intercept of -3 occurs three times the graph of f crosses the x-axis at -3.

The x-intercept of 2 occurs twice - the graph of f touches the x-axis at 2.

$f(x) = -(x+3)^3(x-2)^2$ can also be written as $f(x) = -x^5 - 5x^4 + 5x^3 + 45x^2 - 108$

Degree is _____ which is **ODD** **EVEN**

Leading Coefficient is _____ which is **POSITIVE** **NEGATIVE**

End behavior: as $x \to -\infty$, $f(x) \to$ _____ ∞ $-\infty$ as $x \to \infty$, $f(x) \to$ _____ ∞ $-\infty$

Left-hand side f goes ↑ f goes ↓ *Right-hand side* f goes ↑ f goes ↓

From these six polynomial functions, some generalizations can be made. These generalizations will in fact hold true for all polynomial functions:

If f is of **even degree** the end behaviors are _____ (THE SAME, OPPOSITE).
If the leading coefficient for an even degree function is **positive**,

the function goes _____ (UP, DOWN) on the left-hand side,

while on the right-hand side the function goes _____ (UP, DOWN).
If the leading coefficient for an even degree function is **negative**,

the function goes _____ (UP, DOWN) on the left-hand side,

while on the right-hand side the function goes _____ (UP, DOWN).

If f is of **odd degree** the end behaviors are _____ (THE SAME, OPPOSITE).
If the leading coefficient for an odd degree function is **positive**,

the function goes _____ (UP, DOWN) on the left-hand side,

while on the right-hand side the function goes _____ (UP, DOWN).
If the leading coefficient for an odd degree function is **negative**,

the function goes _____ (UP, DOWN) on the left-hand side,

while on the right-hand side the function goes _____ (UP, DOWN).

If an x-intercept occurs an **odd** number of times,

the graph of f _____ (TOUCHES, CROSSES) the x-axis at that point.
If an x-intercept occurs an **even** number of times,

the graph of f _____ (TOUCHES, CROSSES) the x-axis at that point.

A polynomial function of degree n can have **at most *n - 1* turning points**.

A polynomial function of degree n can **at most *n* x-intercepts**.
If the function is of **even** degree, what is the fewest number of x-intercepts possible?_____

If the function is of **odd** degree, what is the fewest number of x-intercepts possible? _____

To easily remember these generalizations about **end behavior** of polynomial functions, consider the simplest odd degree and even degree functions studied this semester,

Odd degree Degree 1 $f(x) = mx + b$.

Sketch a graph of f when m is positive

Now sketch a graph of f when m is negative

Look at the end behaviors, do they match the generalizations from above

Even degree Degree 2 $f(x) = ax^2 + bx + c$

Sketch a graph of f when a is positive

Now sketch a graph of f when a is negative

Look at the end behaviors, do they match the generalizations from above

Consider the function $f(x) = -4x^7 + 3x^6 + x^5 - 2x^3 + 9x^2 - 11$.

7. What is the degree of this function? _____ Is this an even or odd degree? _____
8. Would the left-hand and right-hand behaviors be the same or opposite? _____
9. What is the leading coefficient? _____ Is this positive or negative? _____
10. The function goes (up or down) _____ on the left-hand side; as $x \to -\infty$, $f(x) \to$ _____
    The function goes (up or down) _____ on the right-hand side; as $x \to \infty$, $f(x) \to$ _____
11. What is the maximum number of x-intercepts this function could have? _____
12. What is the minimum number of x-intercepts this function could have? _____
13. What is the maximum number of turning points this function could have? _____
14. What is the y-intercept of this function? _____

Consider the function $f(x) = 9x^6 + x^5 - 2x^4 + 4x^3 - 13x^2 + x + 2$

15. What is the degree of this function? _____ Is an even or odd degree? _____
16. Would the left-hand and right-hand behaviors be the same or opposite? _____
17. What is the leading coefficient? _____ Is this positive or negative? _____
18. The function goes (up or down) _____ on the left-hand side; as $x \to -\infty$, $f(x) \to$ _____
    The function goes (up or down) _____ on the right-hand side; as $x \to \infty$, $f(x) \to$ _____
19. What is the maximum number of x-intercepts this function could have? _____
20. What is the minimum number of x-intercepts this function could have? _____
21. What is the maximum number of turning points this function could have? _____
22. What is the y-intercept of this function? _____

Consider the function $f(x) = x(x - 7)^4(x + 3)(x + 12)^2 (x - 2)^5$

23. What is the degree of this function? _____ Is an even or odd degree? _____
24. Would the left-hand and right-hand behaviors be the same or opposite? _____
25. Is the leading coefficient positive or negative? _____
26. The function goes (up or down) _____ on the left-hand side; as $x \to -\infty$, $f(x) \to$ _____
    The function goes (up or down) _____ on the right-hand side; as $x \to \infty$, $f(x) \to$ _____
27. What is the maximum number of turning points this function could have? _____

(Continues on next page with problem 28)

28. What are the x-intercepts and their multiplicities? Does the graph touch or cross the x-axis at the x-intercept?

x-int._______, multiplicity ________, Is multiplicity odd or even?________,

Does the graph touch or cross the x-axis at this x-intercept?________

x-int._______, multiplicity ________, Is multiplicity odd or even?________,

Does the graph touch or cross the x-axis at this x-intercept?________

x-int._______, multiplicity ________, Is multiplicity odd or even?________,

Does the graph touch or cross the x-axis at this x-intercept?________

x-int._______, multiplicity ________, Is multiplicity odd or even?________,

Does the graph touch or cross the x-axis at this x-intercept?________

x-int._______, multiplicity ________, Is multiplicity odd or even?________,

Does the graph touch or cross the x-axis at this x-intercept?________

Consider a function of degree 5 whose only zeroes are -1, 3, and 5.

29. Suppose this function crosses the x-axis only at 3 and has a negative y-intercept. Sketch a possible graph for this function.

Since it is of degree 5 we know the end behaviors will be __________ (THE SAME, OPPOSITE).
Since it crosses only at x =3 it will _________ *at x = -1 and x = 5.*
Since the y-intercept is negative it must cross the y-axis at some negative value (below the x-axis).

30. Write a possible factored form of this function. ____________________________________

The x-intercept 3 must be of _________ (EVEN, ODD) *multiplicity,*
while the x-intercepts -1 and 5 must be of _____ (EVEN, ODD) *multiplicity.*

31. Sketch a possible graph for a function which is of degree 3 and whose turning points are (-2, 1) and (6, 8).

32. Sketch a possible graph for a function whose leading coefficient is negative and whose only zeroes are -1 (multiplicity of 2), 5 (multiplicity 3) and 3 (multiplicity 4).

33. Write the fully factored form of $f(x) = x^4 + 8x^3 + 15x^2 - 4x - 20$

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